Paired Samples

Lecture 37
Sections 11.1, 11.2, 11.3

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Mon, Apr 2, 2012

Outline

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Dependent Samples (Paired Data)

- Let the pairs be denoted (x_1, x_2).
- Let $d = x_2 - x_1$.
- We will study the case where d has a normal distribution.
- Let μ_D denote the mean of this distribution and σ_D denote the standard deviation.

Hypothesis Tests Concerning μ_D

- The only null hypothesis for μ_D that we will consider is

$$H_0 : \mu_D = 0.$$

- We will consider any of the three alternatives
  $H_1 : \mu_D < 0.$
  $H_1 : \mu_D > 0.$
  $H_1 : \mu_D \neq 0.$

Hypothesis Tests Concerning μ_D

- For large samples, the test statistic is

$$t = \frac{\overline{d} - 0}{s_D/\sqrt{n}}.$$

(or z)

- For small samples it is necessary that d have a normal distribution. Then the test statistic is

$$t = \frac{\overline{d} - 0}{s_D/\sqrt{n}}.$$

Hypothesis Tests Concerning μ_D

Example (Hypothesis Tests Concerning μ_D)

- Suppose that a group of 10 students take a math placement test.
- Let the variable x_1 represent their scores on that test.
- Then they are given an Algebra refresher course and they retake the placement test.
- Let the variable x_2 represent their scores on the retest.

Hypothesis Tests Concerning μ_D

Example (Hypothesis Tests Concerning μ_D)

- The following table shows the results

Student	1st Score (x_1)	2nd Score (x_2)	Difference (d)
1	83	81	
2	62	63	
3	80	76	
4	73	80	
5	68	78	
6	67	71	
7	68	69	
8	69	78	
9	80	88	
10	83	79	

Hypothesis Tests Concerning μ_D

Example (Hypothesis Tests Concerning μ_D)

- The following table shows the results

Student	1st Score (x_1)	2nd Score (x_2)	Difference (d)
1	83	81	-2
2	62	63	1
3	80	76	-4
4	73	80	7
5	68	78	10
6	67	71	4
7	68	69	1
8	69	78	9
9	80	88	8
10	83	79	-4

Hypothesis Tests Concerning μ_D

Example (Hypothesis Tests Concerning μ_D)

- Test the hypothesis, at the 10% level, that the refresher course improved their grades on the placement test.

Hypothesis Tests Concerning μ_D

Example (Hypothesis Tests Concerning μ_D)

(1) Let x_1 be the first test score, let x_2 be the second test score, and let $d = x_2 - x_1$.
Then the hypotheses are

$$H_0 : \mu_D = 0.$$
$$H_1 : \mu_D > 0.$$

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(2) $\alpha = 0.10$.

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Then the hypotheses are

$$H_0 : \mu_D = 0.$$
$$H_1 : \mu_D > 0.$$

(2) $\alpha = 0.10$.

(3) Let $t = \dfrac{\overline{d} - 0}{s_D/\sqrt{n}}$.

Hypothesis Tests Concerning μ_D

Example (Hypothesis Tests Concerning μ_D)

(4) Compute the value of the test statistic.

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- Enter the x_1 values into $\texttt{L}_1$ and the x_2 values into $\texttt{L}_2$.

Hypothesis Tests Concerning μ_D

Example (Hypothesis Tests Concerning μ_D)

(4) Compute the value of the test statistic.

- Enter the x_1 values into $\texttt{L}_1$ and the x_2 values into $\texttt{L}_2$.
- Evaluate the difference $\texttt{L}_2 - \texttt{L}_1$ and store it in $\texttt{L}_3$.

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- Use `1-Var Stats` $\texttt{L}_3$ to get $\overline{d}$ and s_D.

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- Enter the x_1 values into $\texttt{L}_1$ and the x_2 values into $\texttt{L}_2$.
- Evaluate the difference $\texttt{L}_2 - \texttt{L}_1$ and store it in $\texttt{L}_3$.
- Use `1-Var Stats` $\texttt{L}_3$ to get $\overline{d}$ and s_D.
- We find that $\overline{d} = 3$ and $s_D = 5.354$.

Hypothesis Tests Concerning μ_D

Example (Hypothesis Tests Concerning μ_D)

(4) Compute the value of the test statistic.

- Enter the x_1 values into $\texttt{L}_1$ and the x_2 values into $\texttt{L}_2$.
- Evaluate the difference $\texttt{L}_2 - \texttt{L}_1$ and store it in $\texttt{L}_3$.
- Use `1-Var Stats L3` to get $\overline{d}$ and s_D.
- We find that $\overline{d} = 3$ and $s_D = 5.354$.
- Then

$$t = \frac{3}{5.354/\sqrt{10}} = \frac{3}{1.693} = 1.772.$$

Hypothesis Tests Concerning μ_D

Example (Hypothesis Tests Concerning μ_D)

(5) p-value $=$ `tcdf(1.772,E99,9)` $=$ 0.0551.

Hypothesis Tests Concerning μ_D

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(5) p-value $=$ `tcdf(1.772,E99,9)` $=$ 0.0551.

(6) Reject H_0.

Hypothesis Tests Concerning μ_D

Example (Hypothesis Tests Concerning μ_D)

(5) p-value $=$ `tcdf(1.772,E99,9)` $= 0.0551$.

(6) Reject H_0.

(7) Students scores on the placement test are higher after taking the Algebra refresher course.

Quiz Average vs. Test Average

Example (Quiz Average vs. Test Average)

- Is a student's quiz average higher than his test average, on the average?
- Here are the data from a past semester.

Test Avg.	Quiz Avg.	Diff.
83	59	
89	98	
89	97	
69	58	
71	83	
58	35	
29	53	
55	57	
78	65	
44	24	
66	44	
96	97	
72	92	
89	93	

Quiz Average vs. Test Average

Example (Quiz Average vs. Test Average)

- Is a student's quiz average higher than his test average, on the average?
- Here are the data from a past semester.

Test Avg.	Quiz Avg.	Diff.
83	59	-24
89	98	9
89	97	8
69	58	-11
71	83	12
58	35	-23
29	53	24
55	57	2
78	65	-13
44	24	-20
66	44	-22
96	97	1
72	92	20
89	93	4

Outline

Assignment

Homework

- Read Sections 11.1, 11.2, 11.3, pages 669 - 688.
- Let's Do It! 11.1, 11.2, 11.3.
- Exercises 1 - 8, page 676.
- Exercises 9 -14, page 689.